

The Chain Rule

1. Warm-up problem: A clown is inflating a spherical balloon so that its radius at time t is $\ln(1 + t)$. Find the rate at which the volume of the balloon is changing at time t . (Remember that the volume of a sphere of radius x is $\frac{4}{3}\pi x^3$.)

2. An ant is walking around on the blackboard. The temperature on the blackboard at the point (x, y) is x^4y^2 . The ant's position at time t is given by the vector-valued function $\vec{r}(t) = \langle \cos t, e^t \rangle$. What is the rate of change of temperature experienced by the ant (with respect to time) at any time t ?

3. Quick gradient practice: Find the gradient ∇f of the following functions f .
 - (a) $f(x, y) = x^2 + y^2$.
 - (b) $f(x, y, z) = x^2 + y^2 + z^2$.
 - (c) $f(x, y) = xy$.
 - (d) $f(x, y, z) = xyz$.

4. A fly is flying around a room; his position at time t is $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$. The temperature in the room is given by the function $f(x, y, z) = xyz$. What is the rate of change of the temperature experienced by the fly at time t ?

5. Suppose $z = x^3 + xy + \cos y$, $x = t^2$, and $y = e^t$. Find $\frac{dz}{dt}$.

6. Suppose $u = x^2 + y^2 + z^2$, $x = s^2$, $y = \sin s$, and $z = e^s$. Find $\frac{du}{ds}$.

7. Suppose $z = x^2 - y^2$, $x = \sin st$, and $y = te^s$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

8. (Implicit differentiation.) The equation $x^2y^2 + y^2z^2 + x^2z^2 = 9$ describes the surface shown. Find $\frac{\partial z}{\partial x}$ at the point $(-1, 1, -2)$.

